Microeconomics Comprehensive Exam

June 2007

Instructions:

(1) Please answer each of the four questions on separate pieces of paper.
(2) When you finish, please arrange your answers in the order in which they appear in the questions, i.e., 1 a., 1 b., etc.

1. Consider the following game between two rational, expected-utility maximizing primates, Little Lemur and Big Baboon. There are two bananas, a delicious yellow one and a tasty-but-not-quite-delicious green one. First, Little Lemur picks one of the two bananas. Then Big Baboon, after observing Little Lemur's choice, chooses whether to whack Little Lemur on the head and eat the banana that he picked (in which case Little Lemur is left to eat the other banana), or to eat the other banana and leave Little Lemur alone to eat the banana that he picked. Eating the yellow banana gives either primate 2 units of utility, and eating the green one gives 1 unit of utility. Whacking Little Lemur on the head gives Big Baboon an extra X units of utility, while being whacked on the head costs Little Lemur Y units of utility; both X and Y are strictly positive. The structure of the game is common knowledge.

a. Draw the extensive form of this game.
b. Describe all the subgame perfect equilibria of this game, assuming that $X > 1$.
c. Describe all the subgame perfect equilibria of this game, assuming that $X < 1$.
d. Is Little Lemur better off when Big Baboon really likes whacking him ($X > 1$) or when she doesn't like it that much ($X < 1$)? Explain.
e. Now suppose that it is common knowledge that Big Baboon, like many of the great thinkers of our age, is green-yellow colorblind, and so CANNOT TELL which banana Little Lemur picked. Draw the extensive form of this new game, and find all the perfect Bayesian equilibria, assuming that $X < 1$.

2. Dee Sider, a risk-neutral expected-gold maximizer, must choose between two technologies for turning lead into gold. He has one divisible unit of lead. There is no discounting. Technology A is straightforward: each unit of lead is immediately transformed into one unit of gold. Technology B is more complicated: the lead must be fed into the machine at a constant rate of 1 unit of lead per unit of time. With probability $p \in (0, 1)$ the technology succeeds, and each unit of lead is transformed into $K > 1$ units of gold. With probability $1 - p$, the technology fails, and no gold is produced. It is not apparent whether the technology succeeds or fails until a random time t; t is distributed uniformly on $[0, 1]$. If the technology fails, then all lead fed into the machine up until time t is lost.

Dee's problem, then, is as follows. He chooses a stopping time $s \in [0, 1]$. He feeds lead into Technology B up until $\min\{s, t\}$. If $s < t$, then at time s he abandons Technology A (losing the s units of lead already fed into it), feeds the remaining $1 - s$ units of lead into Technology A, and winds up with $1 - s$ units of gold. If $t \leq s$ and Technology B succeeds, then he continues to feed lead into Technology B and winds up with K units of gold. If $t \leq s$ and Technology B fails, then at time t he feeds the remaining $1 - t$ units of lead into Technology A and winds up with $1 - t$ units of gold.

a. What is Dee's expected payoff (in units of gold) as a function of s?
b. What is the optimal stopping time correspondence $s^*(p, K)$? (HINT: check second-order conditions.)
c. Will Dee ever choose to start with Technology B (i.e., $s^* > 0$) when $pK < 1$? Explain. (HINT: why is $pK < 1$ an interesting condition?)

3. Consider an industry in which there are n identical profit-maximizing firms. The i^{th} firm produces output y_i using a single input $x_i \geq 0$. The production function is given by

$$y_i = \frac{x_i}{x} f(x),$$

where $x = \sum_{j=1}^{n} x_j$ is the sum of the inputs x_j used by the individual firms, as long as $x_j > 0$ for some j. If $x = 0$, then nobody produces, and $y_i = 0, \forall i$. Assume that f is continuous, $f(0) = 0$, $f'(x) > 0$ for all $x > 0$, $f''(x) < 0$ for all $x > 0$, $\lim_{x \downarrow 0} f'(x) = \infty$, and $\lim_{x \to \infty} f'(x) = 0$. The firms take all prices as given; the price of the output is $p = 1$, and the price of the input is $w > 0$. Assume that each firm treats the actions of other firms as given in making its decisions about production.

a. Find a symmetric Nash equilibrium in this industry.
b. Deduce that in the equilibrium all firms make positive profits.
c. What happens to this industry in the long run if free entry is allowed?
d. In the long run (with free entry), does the industry approach Pareto efficiency? Why or why not?

4. Consider an economy with m agents and 2 goods, x and g. Suppose that the first good, x, is a private good and the second good, g, is public. Agents' private good consumption sets are identified with $\mathbf{R}_+$, and utility functions, $u_i(x_i, g)$, are continuously differentiable in both arguments, strictly increasing, and strictly concave. Consumer i has an initial endowment of $e_i > 0$ units of the private good, and there is a technology to transform the private good into the public good: $g = y(x)$.

a. Suppose that there are n competitive firms that have access to the public good provision technology, and that consumers own equal shares of each firm. How would you define a competitive equilibrium in this economy?

For the remainder of the question suppose that the public goods provision technology exhibits constant returns to scale, such that two units of the private good can be converted into one unit of the public good.

b. What is the equilibrium price ratio?
c. Are competitive allocations Pareto efficient?
d. Relate your answer to question 4.c to the First Basic Welfare Theorem.