Microeconomics Comprehensive Exam

June 2008

Instructions:
(1) Please answer each of the four questions on separate pieces of paper.
(2) When finished, please arrange your answers alphabetically
(in the order in which they appeared in the questions, i.e. 1 a., 1 b. etc.).

1. Consider the following infinitely-repeated "risk-sharing" game. There are N ex-ante identical players, each with twice-continuously-differentiable, bounded, and strictly concave Bernoulli utility function u: $\mathbf{R}_+ \rightarrow \mathbf{R}$. In each period t, player i receives random income x_{it}, drawn according to cdf $F(x)$, which has continuous density $f(x)$ on support $[x_L, x_H] \subseteq \mathbf{R}_+$. Income is i.i.d across time and across players. Within a period, after incomes are realized, players can (simultaneously) transfer some or all of their incomes to another player or players. There is no storage – any period-t income not consumed by some player in period t is lost. All players have the same discount factor $\delta \in (0, 1)$. The structure of the game is common knowledge.

The following notation may be useful: Let $\bar{x}_t = \frac{\sum_{i=1}^{N} x_{it}}{N}$ denote the average income across players in period t, and let $\bar{F}$ and $\bar{f}$ denote, respectively, the associated cdf and pdf.

For parts a and b, suppose that in a period each player's realized income is observed by all players.

a. Describe the strategy space for a player in the stage game.
b. Find the values of the discount factor δ at which the following outcome is achievable in subgame perfect equilibrium: in each period, each player consumes $\bar{x}_t$ (that is, player share income equally) as long as all players have shared equally in all previous periods. Otherwise, each player consumes his own income x_{it}.

For parts c and d, suppose that player i observes only his own income x_{it}, and not the income of any other player.

c. Describe the strategy space for a player in the stage game.
d. Find the values of the discount factor δ at which the outcome described in part b is achievable in a sequential equilibrium.

2. Amalgamated Kittens, Inc. (AKI) produces a single output good (kittens) using two inputs (food and love), according to the production function $\Phi(f,l) = f^a l^b$, where $a,b \in (0,1)$. There is free disposal. The strictly positive prices of kittens and food are respectively p_k and p_f. Love cannot be bought. Instead, the amount of love available to be used in production is proportional to the quantity of kittens produced: $l = gk$, where $g > 0$ is a scalar.

a. Write out AKI's profit maximization problem and the associated first-order conditions. DO NOT solve them.
b. Derive AKI's supply correspondence, assuming that $a < 1-b$.
c. Derive AKI's supply correspondence, assuming that $a > 1-b$.

3. Consider a two-period economy under uncertainty composed of a continuum of workers/consumers, the total mass of population being normalized to one. There is no consumption at date $t = 0$. There is a single consumption good produced by a single input (labor). Production and consumption takes place at $t = 1$. All workers are identical *ex ante*. Each of them has a probability $\frac{1}{2}$ of being able to work at $t = 1$, in which case a worker produces k units of consumption good at no cost. With probability $\frac{1}{2}$, a worker is disabled at $t = 1$, and produces nothing. The probability of disability is independent across workers. The state of each worker is public information after the uncertainty had been realized. The utility of consumption of an amount c is $u_a(c)$ if the worker is capable of producing and $u_d(c)$ if she is disabled. Assume that u_a, u_d are of the class C^2, increasing, strictly concave, and satisfy the Inada conditions (i.e., $u_{a,d}(0) = 0$, $\lim_{c\to\infty} u_{a,d}(c) = \infty$, $\lim_{c\to 0} u'_{a,d}(c) = \infty$, $\lim_{c\to\infty} u'_{a,d}(c) = 0$). Further assume that for any $c > 0$, $u_a(c) < u_d(c)$.

(a) Suppose there is a full set of Arrow-Debreu markets which are open at $t = 0$. Carefully define a competitive equilibrium and find an equilibrium[1].

(b) Now suppose that the state of each worker is private information so that a worker can claim being disabled and stop working. Assume that there is a market for insurance represented by a competitive insurance company that makes a take-it-or-leave-it offer (T, q) to a worker, where T is the payment a worker receives if she is disabled, and q is the insurance fee (this fee is paid by an insured worker at $t = 1$ no matter what her state is). The contract is signed at $t = 0$, and it is enforceable. Both T and q are in units of consumption good. Define a competitive equilibrium for this economy.

Let the utility functions satisfy the following condition:

$$u'_a(c_a) = u'_b(c_b) \quad \Rightarrow \quad c_a > c_d.$$

Show that the equilibrium allocation is the same as in part (a).

(c) Let the environment be the same as in part (b) except that now, the utility functions satisfy the following condition:

$$u'_a(c_a) = u'_b(c_b) \quad \Rightarrow \quad c_d > c_a.$$

Show that the equilibrium allocation that you derived in part (b) cannot be reached. Find an equilibrium allocation.

[1] Don't forget that the workers are identical *ex ante*!!

4. Consider the following two-agent economy with an externality. Alice is the agent who produces the externality measured by the level $h \geq 0$, and the externality affects Bob. The agents' value functions are quasi-linear in wealth, so that

$$\begin{aligned} v_a(h; w_a) &= \phi_a(h) + w_a, \\ v_b(h; w_a) &= k\phi_b(h) + w_b, \end{aligned}$$

where $k \in \{k_L, k_H\}$ is a random variable whose realization is Bob's private information. Assume that $0 < k_L < k_H$; prob$(k = k_L) = p \in (0,1)$, prob$(k = k_H) = 1 - p$. The probability distribution is common knowledge. Also assume that (i) ϕ_a, ϕ_b are of class C^2; (ii) $\phi_b(0) = 0$, $\phi_b' < 0$ for all $h > 0$; (iii) $\lim_{h \to 0} \phi_a'(0) = \infty$; (iv) $\phi_i'' < 0\, (i = \{a, b\})$, (v) there exists h^* s.t. $\phi_a'(h^*) = 0$.

(a) What level of externality will be produced in this economy?

(b) Which levels of externality are efficient if k is public information?

(c) Let Bob have the right to live in the environment which is free of the externality. Assume that the agents are allowed to bargain over the level of externality after the realization of k has been observed by Bob. Let Alice have the full bargaining power. What will be the levels of externality produced, and what will be the agents' surpluses?

(d) Suppose that the government mandates that the level of externality cannot be higher than the *ex ante* efficient level. How your answers to (c) will change?

(e) Is the government intervention beneficial? Why or why not?