We discuss economic aggregation and political aggregation in the context of a simple dynamic version of the canonical political-economy model—the Meltzer-Richard model. Consumers differ both in labor productivity and initial asset wealth and there is no physical capital. Under commitment, and for preferences that imply aggregation in assets and productivity, the induced policy preferences do not depend on any distributional characteristics other than means, but the median-voter theorem does not apply other than in special cases. Without commitment, a median-voter theorem is easier to obtain, but the induced policy preferences depend on distributional characteristics.
1 Introduction

In this paper we develop a simple dynamic version of the canonical political-economy model—the Meltzer-Richard model—in order to obtain some analytical characterization of policy determination. There is no capital, but consumers can trade intertemporally in an asset market; the economy is closed, and the equilibrium determines the interest rate. Consumers potentially differ in two dimensions: they have different initial asset holdings, which determines whether they will be asset-rich or asset-poor, and they have different labor productivity, which will determine their leisure choice and their labor income. Policy amounts to a proportional tax on labor income and a lump-sum transfer subject to period-by-period budget balance. Intuitively, the taxation of labor income has two effects that both imply some form of redistribution: first, it redistributes between consumers with different amounts of labor income (because they have different productivity or because, if there are differences in wealth and there is a wealth effect on labor supply, they work different amount of hours); and second, to the extent taxation varies over time, it influences the rate of interest, which affects consumers with different asset holdings differently.

We show that for a general class of utility functions, if dictatorial power is given to any agent to determine, with commitment, an infinite policy sequence at time 0—a “commitment equilibrium”—this sequence will feature a constant tax-transfer from the second period and on. Thus, our fully dynamic economy reduces to a two-dimensional policy choice: the tax rate at zero and the tax rate thereafter. If there is no initial asset heterogeneity—a repeated version of the Meltzer-Richard model—and preferences admit aggregation, there is also (median-voter) political aggregation; moreover, the tax-transfer will be constant from time zero and on and the commitment equilibrium will be time-consistent. In contrast, if there is asset heterogeneity but no heterogeneity in labor productivity, then the commitment equilibrium is time-inconsistent; if, moreover, preferences admit aggregation, taxes will be zero from the second period and on. In the latter case, there is political aggregation as well, and a median-voter theorem applies. With two-dimensional heterogeneity and preferences that admit aggregation (over both assets and productivity differences), the induced policy preferences over the two tax rates can be derived independently of the distribution of types, but political aggregation—say, a median-voter theorem—typically does not obtain. Under lack of commitment—in a Markov-perfect equilibrium where the tax rate is determined period by period—policy preferences generally depend on the distribution of types, but political aggregation obtains much more easily.

The previous literature extending Meltzer and Richard (1981) to dynamic settings includes Krusell and Ríos-Rull (1999), which considers a model like the one presented here but with a neoclassical capital-accumulation technology. There, the sole focus is on Markov-perfect equilibria, and a one-period implementation lag for policy is employed. An intermediate case between that paper and the present paper is Azzimonti, de Francisco, and Krusell (2006), which focuses on the role of economic aggregation when there is capital accumulation but no commitment. Finally, Azzimonti, de Francisco, and Krusell (2007) study the special case of the present model without heterogeneity in productivity both with and without commitment.
2 Environment

In this infinite horizon economy there is no uncertainty and time is discrete. Agents value consumption, $c$, and leisure, $l$, and discount the future at rate $\beta$:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t).$$

(1)

Agents can trade one-period bonds (which are in zero net supply), each promising one unit of consumption next period. The heterogeneity within the population enters in two ways: (i) through differences in consumers’ initial asset holdings, $a_0$; and (ii) through labor-productivity differences, $\theta$.

The identity of an agent is determined by the pair $(a_0, \theta)$, which is drawn at the beginning of life from a distribution $F(a_0, \theta)$ with finite support. We denote the conditional expectations $E(a_0/\theta) = \bar{a}$ and $E(\theta/a_0) = \bar{\theta}$ and the measure of each type with $\mu_i$ for $i = \{1, ..., I\}$. Population size is normalized to one: $\sum_i \mu_i = 1$.

Production takes place according to a stationary production function which depends linearly on labor. Output is used for consumption, and there is no government consumption or investment. We also abstract from capital entirely. Each consumer has one unit of time, so that $1 - l^i$ denotes the number of hours worked. We make assumptions on primitives so that agents’ decision problems are strictly concave; hence, all agents of the same type will make the same decisions.

3 Equilibrium and economic aggregation

There are two prices determined in equilibrium: the price of one-period bonds, denoted by $q_t$, and the wage rate, $w_t$, both measured in terms of consumption goods in the same period. In addition, there is a government that taxes labor income at a proportional rate $\tau_t$ in period $t$ and makes equal lump-sum transfers $T_t$ back to all consumers under a balanced budget. Thus, a typical consumer budget constraint in period $t$ reads

$$c^i_t + q_t a^i_{t+1} = a^i_t + \theta^i w_t (1 - l^i_t)(1 - \tau_t) + T_t.$$ 

(2)

In equilibrium, consumers’ holdings of assets have to add to zero: $\sum_i \mu_i a^i_t = 0$, where $a^i_t$ denotes the total holdings of agents of type $i$. We define a competitive equilibrium for a given sequence of government policy as follows:

Definition 1 Given a tax policy $\{\tau_t\}_{t=0}^{\infty}$, a competitive equilibrium is a sequence of prices $\{w_t, q_t\}_{t=0}^{\infty}$ together with a sequence of allocations $\{\{c^i_t, a^i_{t+1}, l^i_t\}_{t=1}^{\infty}, T_t\}_{t=0}^{\infty}$ satisfying the following conditions:

1. For all $i$, $\{c^i_t, a^i_{t+1}, l^i_t\}_{t=0}^{\infty}$ maximizes (1), subject to (2) for all $t \geq 0$, with $a_0 = A_{i0}$.

2. For all $t \geq 0$, $w_t = 1$ and $q_t$ is such that $\sum_i \mu_i a^i_t = 0$.

3. For all $t \geq 0$, $T_t = \tau_t \sum_i \mu_i \theta^i (1 - l^i_t)$.
We will (implicitly) employ utility functions and constraints implying interior solutions for all consumers. Agent $i$ will choose to accumulate assets and consume leisure so that

$$u_c(c_i^t, l_i^t)w_i\theta^t(1 - \tau_t) = u_l(c_i^t, l_i^t) \quad \text{and}$$

$$q_tu_c(c_i^t, l_i^t) = \beta u_c(c_{i+1}^t, l_{i+1}^t).$$

Together with the budget constraints and a transversality condition, these conditions are sufficient for finding optimal consumer choices. In general, aggregate allocations and prices depend on the joint distribution of asset holdings and productivity levels, $F(a_0, \theta)$. Under some additional assumptions on preferences it is possible to show that aggregates and prices depend only on two moments: average asset holdings (zero, in equilibrium) and average productivity, $\bar{\theta}$. Because the ratio between consumption and leisure of any individual becomes linear in productivity $\theta$ it is straightforward to verify the following:

**Proposition 2** Under utility functions $u(c, l)$ of the CRRA form: $g(c, l) = \frac{c^{1-\sigma}l^\sigma}{1-\alpha}$, with $g(c, l) = c^{\alpha}l^{1-\alpha}$ and $\alpha, \sigma > 0$, the competitive equilibrium exhibits aggregation: prices and aggregate allocations depend only on the first moments of the distribution over assets and labor productivity.

Under logarithmic utility the competitive equilibrium can be fully characterized in closed form, as stated in the corollary below:

**Corollary 3** When $\sigma = 1$, so $u(c, l) = \alpha \ln c + (1 - \alpha) \ln l$, the competitive equilibrium can be characterized by the following allocations and prices:

$$\bar{C}_t = \bar{\theta}\alpha \frac{1 - \tau_t}{1 - \alpha \tau_t}, \quad c_i^t = \gamma(\tau, \theta^t)C_i, \quad l_i^t = \frac{(1-\alpha)c_i^t}{\alpha \theta^t (1 - \tau_t)}, \quad \text{and} \quad q_t = \beta \frac{1 - \tau_t}{1 - \alpha \tau_t} \frac{1 - \sigma \tau_t}{1 - \tau_t}.$$  

where

$$\gamma(\tau, \theta^t) = \frac{\beta}{\bar{\theta}} \left[ \frac{1 - \alpha \tau_t}{1 - \alpha \tau_t - \beta \tau_t} \right].$$

When the only dimension of heterogeneity is given by differences in initial asset holdings (so that all agents have the same productivity), more general aggregation results can be derived. For example, Azzimonti, de Francisco and Krusell (2007) show that aggregation, in the sense defined above, holds for any homogeneous of degree one function $g(c, l)$, such as the CES function $(\alpha c^\rho + (1 - \alpha) l^\rho)^{\frac{1}{\rho}}$. It is also possible to obtain aggregation results where prices and aggregate quantities depend on a small set of other moments of the distribution; see, e.g., Correia (2006).

## 4 Optimal policy under commitment

In this section we describe agents’ preferences over a tax sequence assuming that individuals vote over the whole sequence at period zero. That is, we assume that the government can commit to implementing policy in the future. We restrict attention to Ramsey equilibria, which implies that any feasible policy must be consistent with competitive equilibrium allocations. Before looking at voting equilibria, we consider the time-zero tax sequence choice made by an arbitrary consumer $i^*$. 
Definition 4 The tax sequence \( \{\tau_t\}_{t=0}^\infty \) preferred by agent \( i^* \) in a Ramsey Equilibrium is the one that maximizes the lifetime utility as given by (1) for \( i = i^* \) subject to (a) the set of budget constraints, i.e., (2), for all \( t \geq 0 \) and all \( i \); (b) the set of optimality conditions for leisure and for asset holdings, i.e., (3) and (4), respectively, for all \( t \geq 0 \) and all \( i \); and (c) the market-clearing conditions 2-4 in Definition 1 (see eq.(5) in the Appendix for more details).

The resulting maximization problem is not, in general, globally a convex problem. For the remaining discussion, however, we will presume that our primitives are such that it is; in particular, we will presume that first-order conditions are sufficient. In the next proposition, proved in the Appendix, we characterize the resulting time structure of taxes preferred by any agent.

Proposition 5 For any differentiable \( u(c, l) \), a first-order characterization for optimal taxes implies that taxes are constant from period \( t = 1 \) and on. The infinite sequence of taxes preferred by any agent \( i^* \) can thus be represented by a tax pair \( (\tau^*_0, \tau^*_1) \) where \( \tau^*_0 \) is the tax rate in period 0 and \( \tau^*_1 \) is the tax rate from period 1 on.

Thus, our infinite-horizon economy leads to a preferred tax sequence characterized by only two numbers: the tax rate in the first period and the tax rate thereafter. No more than two rates are needed due to the stationarity of the economy and the absence of capital; no less than two rates are needed due to the special role played by the first period, thus illustrating that the commitment solution is not, in general, time-consistent.

4.1 Characterizing the \( (\tau^*_0, \tau^*_1) \) pair using economic aggregation

Under the log-log preferences used in Corollary 3, our aggregation result dictates that the representative agent has \( a_0 = 0 \) and \( \theta = \bar{\theta} \). Furthermore, the indirect utility takes a very simple form, and it is independent of other moments of the distribution \( F(a_0, \theta) \) than mean ability (\( \bar{\theta} \)) and average initial asset holdings, which are zero. As a result, the preferred taxes will also be independent of how wealth and ability are distributed. Agent \( i^* \)'s preferred policy sequence is the one that maximizes his indirect utility, which in general can be shown to equal \( V(\tau, \theta^*) = \sum_i \beta^i \left[ \alpha \ln(1 - \tau_i) - \ln(1 - \alpha \tau_i) + \ln(\gamma(\tau, \theta^*)) \right] \), where \( \tau \) denotes the whole tax sequence; see Corollary 3 for the definition of \( \gamma(\tau, \theta^*) \).

As stated in Proposition 5 for a more general case, the necessary conditions reduce to two equations in two unknowns:

\[
-\bar{\theta} \gamma(\tau^*, \theta^*)(1 - \alpha)\tau^*_1 + (\bar{\theta} - \theta^*)(1 - \tau^*_1)(1 - \alpha \tau^*_1) = 0,
\]

\[
-\bar{\theta} \gamma(\tau^*, \theta^*)(1 - \alpha)\tau^*_0 + (\bar{\theta} - \theta^*)(1 - \tau^*_0)(1 - \alpha \tau^*_0) + \frac{1 - \alpha}{\alpha} \frac{1 - \alpha \tau^*_1}{(1 - \tau^*_0)} \theta^*_0 = 0.
\]

As should be expected, the representative agent prefers \( \tau_0 = \tau_1 = 0 \) (since \( \gamma(\tau, \bar{\theta}) = 1 \)). We will now discuss the characterization of the solution to this equation system by looking first at special cases and then considering the general case.
4.1.1 No asset inequality

Without asset heterogeneity the economy collapses to an infinitely repeated version of the one studied by Meltzer and Richard (1981). This case exhibits no interesting dynamics, as the tax sequence preferred by any agent in the economy is time-invariant. The price of one-period bonds will therefore satisfy $q_t = \beta$: because there is no asset heterogeneity, no agent sees a gain from manipulating $q_t$ by time-varying taxes. It is easy to show that if an agent’s productivity is lower (higher) than average, then a tax on (subsidy to) labor is preferred.

4.1.2 No skill inequality

When productivities do not differ but there is asset inequality, the analysis (which is contained in Azzimonti, de Francisco, and Krusell, 2007) shows that there is active interest-rate manipulation, unless $i^*$ is the representative, zero-asset agent. A tax hike at time zero, for example, raises consumption growth between the first two periods and thus increases the interest rate, which is a benefit for asset-rich agents. A similar hike in the future raises the corresponding interest rate but, however, also lowers the interest rate one period earlier. Taxes, of course, also directly redistribute to the extent agents with different asset holdings work different hours.

Under preferences that admit economic aggregation—which, as pointed out above, is now a larger class—this economy has the striking feature that all agents agree on the tax rate from period 1 and on: it should be zero. A tax hike at $t$ redistributes toward the asset-rich at that date (since they have lower wage income due to a wealth effect on labor supply) but $q_{t-1}$ falls, an effect which operates on all of the income of the asset-rich, and for this class of preferences these two effects offset exactly. As for $\tau^*_0$, the offsetting interest-rate effect is not present, of course, leading the asset-rich to prefer positive time-zero taxes.

4.1.3 The general case

Although there is no closed-form solution for the two tax rates, it is straightforward to compute them numerically by solving the equation system displayed above. Figure 1 illustrates.

![Figure 1: Optimal policy for different types](image-url)
The dashed lines depict the preferred $\tau_1^*$ values for agents with different $a_0$ (on the horizontal axis); different dashed lines correspond to different values of $\theta$. Two conclusions emerge: (i) the future tax rate is positive if and only if $\theta < \bar{\theta}$ ($\bar{\theta}$ is normalized to 1 in the figure); and (ii) the future tax rate preferred by agent $i^*$ barely depends on his initial asset holdings at all. In sum, the future tax rate is dictated by Meltzer-Richard arguments, even though there is asset inequality in the economy.

As for the time-zero tax rate ($\tau_0^*$), its dependence on $a_0$ and $\theta$ is given by the solid lines. The key conclusion here is that time-zero rates are given by a combination of Meltzer-Richard arguments—more productive agents prefer lower tax rates—and those in Azzimonti, de Francisco, and Krusell, (2007): the asset-rich prefer higher tax rates, due to interest-rate effects.

Note, finally, that in the general case, the two first-order conditions are interdependent and the determination of $\tau_1^*$ depends on the choice of $\tau_0^*$; reference to this observation will be made later.

5 Political aggregation

The types of models studied in this paper are complex due to the presence of two dynamic-equilibrium layers: the economic and the political one, and a potentially large state space. Theory related to existence, uniqueness, and characterization of a non-empty core for voting rules is scarce, and there are few theorems that apply with generality. We now provide some results for the present model that are based on assuming that economic aggregation obtains.

5.1 One-dimensional heterogeneity

Under one-dimensional heterogeneity, either in skills or in assets, political aggregation is simple to obtain: the median voter theorem holds as long as indirect preferences are single-peaked or a single crossing condition applies, which one can verify for classes of preferences. The case with skill heterogeneity only is covered in Meltzer and Richard (1981); the case with asset inequality only also admits a median-voter theorem, as long as preferences admit economic aggregation.¹

5.2 Two-dimensional heterogeneity

Two-dimensional heterogeneity need not pose a problem for political aggregation; see, e.g., the analyses in Grandmont (1978) and Persson and Tabellini (2000). However, when there are two policy variables—recall that the induced preferences in our infinite-horizon model are defined over $(\tau_0, \tau_1)$ here—one needs very strong symmetry assumptions on these induced preferences, which will not generally obtain. That is, the restrictions imposed by the underlying economic model generally do not imply that a core exists. As an illustration, Figure 2 shows the induced indifference curve for tax-rate pairs of an agent with logarithmic utility: it is neither elliptical, nor symmetric, around its preferred point.

We will merely make three points here. One is simple: economic aggregation is still useful, because it makes the induced preferences independent of

¹See Azzimonti, de Francisco, and Krusell (2007) for the detailed arguments.
the asset-productivity distribution. I.e., the indifference curves such as those in Figure 2 do not depend on how assets and productivities are distributed and can thus be derived separately.

Second, we will illustrate how a median-voter theorem can be obtained: by placing severe restrictions on the asset and productivity heterogeneity. In general, there is no natural “middle” in the two-dimensional space, and any policy can typically be beaten, say, by a majority. So for illustration suppose that utility is logarithmic and that productivity an initial assets each can take on three values: \( \theta^i \in \{ \bar{\theta} - \epsilon, \bar{\theta}, \bar{\theta} + \epsilon \} \) with \( \epsilon > 0 \) and \( a^i_0 \in \{ -\delta, 0, \delta \} \) with \( \delta > 0 \). Thus, there are 9 types of agents symmetrically distributed around \( a^0_0 = 0 \) and \( \bar{\theta} = 1 \). The left panel of Figure 3 depicts the implied preferred combination of taxes for each type.

In this example, there is a well-defined median agent in the distribution—the mean agent—and we expect that his preferred policy (zero taxes at all times) will beat any alternative tax pair \((\tau_0, \tau_1)\) in pairwise voting. To prove this, we can apply Theorem 5.7 in Austen-Smith and Banks (1999) and its corollaries to show that indeed that is the case, as long as the Plott conditions
are satisfied (which in this context are necessary and sufficient). These conditions require that for any coalition of agents there must exist a ‘pairing’: if any small deviation from the proposed policy pair benefits one agent, it must be detrimental for another agent. Let the gradient of agent $i$ be denoted by $dV_i(\tau^*_m)$.

Mathematically, the condition implies that there must exist a constant $\lambda_{ij}$ such that $dV_i(\tau^*_m) = -\lambda_{ij}dV_j(\tau^*_m)$ for some $\lambda_{ij}$. For example, an agent with $\theta = \bar{\theta}$ and $a_0^i = \delta$ can be paired with an agent with $\theta = \bar{\theta}$ and $a_0^j = -\delta$. One can similarly pair all remaining agents.

In any “asymmetric” case, however, the core typically does not exist. Suppose that the 9 types of agents are instead composed by all possible combinations of $\theta$ and $a_0$ where $\theta \in [0.89, 0.91, 1.2]$ and $a_0 \in [-0.07, -0.03, 0.10]$, implying that the agent with $\theta = 0.91$ and $a_0 = -0.03$ would be median in each dimension separately. Now the median agent’s preferred policy would be beaten by a policy proposed by agent with the same asset level but lower productivity (agent E in the right panel of Figure 3). This policy would not be a majority-voting equilibrium either, however, since it does not satisfy the Plott conditions: the core is empty in this case.

Third and finally, the nonexistence of a core does not make political-economy analysis uninteresting in this model, of course. It just means that outcomes depend on the exact nature of the policy games played. Thus, one must specify further details about these games in order for outcomes to be well defined. Then positive as well as normative properties of different games can be examined and compared.

6 Sequential voting: lack of commitment

The induced preferences over policy under commitment are generally such that $\tau_0$ and $\tau_1$ are different. This means that the commitment equilibrium is not time-consistent: due to the intertemporal character of the asset decision, changes in $\tau_0$ have a different effect than do changes in other $\tau_t$s.

If we employ a Markov-perfect equilibrium concept, as in Azzimonti, de Francisco, and Krusell (2007), the median-voter theorem may well hold even if the distribution of types is asymmetric. This is because agents are voting on a one-dimensional policy at $t$: $\tau_t$. Tax outcomes then tend to look like in the first period of the case with commitment, i.e., they are influenced by asset as well as by productivity heterogeneity. Thus, long-run tax rates depend crucially on whether or not there is commitment.

The Markov-perfect equilibrium is more complex to analyze also because the induced policy preferences depend on the distribution of agents over wealth and productivity. This is because these preferences need to be forward-looking: the current distribution will evolve and influence future policy, which in turn influences the effects of current policy.

One might also note that the one-period “policy implementation lag” used in Krusell and Ríos-Rull (1999), as well as more recently in work by Corbae, D’Erasmo, and Kurusçu (2007), will have a nontrivial effect on outcomes—and might improve welfare—if there is general inequality in both assets and productivity. Now, since the preset tax influences the choice of next period’s tax rate, as seen above, implementation-lag equilibria are also time-inconsistent.

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2In our example $dV(\tau^*_m) = \left( \frac{\alpha(\theta-\bar{\theta})(1-\alpha)a_0}{\alpha\theta(1-\alpha)+\bar{\theta}} : \frac{\alpha(\theta-\bar{\theta})}{\alpha\theta(1-\alpha)+\bar{\theta}} \right)$.

3This means that $\lambda_{ij} = [\theta - \delta(1-\beta)]/[\theta + \delta(1-\beta)]$. 

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9
7 Conclusion

In this paper, we have discussed the relation between economic and political aggregation in the context of a simple dynamic version of the seminal political-economy model—the Meltzer-Richard model—where consumers differ both in productivity and in initial asset wealth. For a general class of utility functions, the sequence of taxes preferred by any agent reduces to a two-dimensional object: a rate for time zero and a rate for all later periods. Under commitment, future taxes are mainly determined by productivity differences (the Meltzer-Richard mechanism), while period-zero taxes are influenced also by wealth inequality. Without commitment, in a Markov-perfect equilibrium, current and future taxes are jointly influenced by wealth and productivity inequality.

8 Appendix

Agent $i^*$ solves

$\mathcal{L} = \max \sum_{t=0}^{\infty} \beta^t \left\{ u(c^t_i, l^t_i) + \sum_i \mu^i \lambda^i_{t1}[-c^t_i + a^t_i - q^t_i a^t_{i+1} + \theta^t w^t(1 - l^t_i)(1 - \tau_t)] + \right.$

$\left. + \tau_t \sum_i \mu^i \theta^t(1 - l^t_i)] + \sum_i \mu^i \lambda^i_{t2} \left[ u(c^t_i, l^t_i) w^t(1 - \tau_t) - u(c^t_i, l^t_i) \right] + \sum_i \mu^i \lambda^i_{t3} \left[ q^t_i u_c(c^t_i, l^t_i) - \beta u_c(c^t_{i+1}, l^t_{i+1}) \right] + \lambda_{t4} \left[ \sum_j \mu^j \theta^j (1 - l^j_i) - \sum_j \mu^j c^j_i \right] \right\}.$

The FOC are, with $I_i = 0 \forall i \neq i^*$ and where $\lambda^i_{t-1,3} = 0$ for $t = 0$, are

$I_i u^t_{c_i} - \lambda^i_{t1} + \lambda^i_{t2}[u^t_{cc}(\theta^t(1 - \tau_t) - u^t_{ct}) + \lambda^i_{t3} q^t_i - \lambda^i_{t-1,3}) - \lambda^i_{t4} = 0 \quad (c^t_i)$

$I_i u^t_{a_i} - \lambda^i_{t1} \theta^t(1 - \tau_t) - \tau_t \theta^t \sum_j \mu^j \lambda^j_{t1} + \lambda^j_{t2}[u^j_{cl}(\theta^j(1 - \tau_t) - u^j_{ct}) + u^j_{ct}(\lambda^j_{t3} q^t_j - \lambda^j_{t-1,3}) - \lambda^j_{t4} \theta^j = 0 \quad (l^t_i)$

$- \sum_j \lambda^j_{t1} \mu^j a^j_{t+1} + \sum_j \lambda^j_{t3} \mu^j w^j = 0 \quad (a^t_{i+1})$

$\lambda_{t1} q^t_i - \lambda_{t+1,1} \theta^t = 0 \quad (q^t_i)$

$\sum_i \lambda^i_{t1} \mu^i \theta^i (1 - l^t_i) + \left[ \sum_j \mu^j \theta^j (1 - l^j_i) \right] \sum_i \mu^i \lambda^i_{t1} - \sum_i \lambda^i_{t3} \mu^i u^i_{ct} \theta^i = 0 \quad (\tau_t)$

We need to show that policy can be summarized by $(\tau_0, \tau_1)$ and allocations, prices, and multipliers that are constant from $t \geq 1$. The unknowns to be solved for are: 6I multipliers related to individual choices, $\{\lambda^i_{t1}, \lambda^i_{t2}, \lambda^i_{t3}\}_{i,t}$ for $t = 0,1$ and $\forall i$, 2 multipliers associated to market clearing conditions, $(\lambda_{t4}, \lambda_{t4})$, 5I individual allocations $\{c^t_i, l^t_i, a^t_i\}_{i,t}$, 2 prices $(q_0, q_1)$, and 2 tax levels $(\tau_0, \tau_1)$, i.e., a total of 11I+6 unknowns. Under the guess, the equations at
$t > 1$ are identical (to be verified later). The FOCs at 0 and 1 are different in that: (a) they involve initial conditions, $a_0^i$, and (b) the FOCs for $c_t^i$ and $l_t^i$ involve multipliers in the previous periods (i.e., $\lambda_{t-1,3}$). For periods 0 and 1, there are $6I+2$ FOCs for the multipliers, 2 for prices, 2 for taxes and 6I for individual allocations, totalling 12I+6 equations. Under the guess, $c_t^i$ is constant and $q_t = \beta$ for $t \geq 1$, so $I$ FOCs for $\lambda_{1,3}$ cancel out (those involving the $a_2^i$ choices). This leaves $5I+2$ linearly independent equations that can be used to solve for competitive allocations and prices ($5I+2$ unknowns). Knowing these, we can find $6I+2$ multipliers and 2 tax levels from the remaining equations (a total of $6I + 4$ unknowns). There are $5I + 4$ independent equations obtained from the FOCs for allocations, policies, and prices (note that the FOC for $a_2^i$ does not add new information under the guess). We are then missing $I$ equations to complete the system. In addition, we need to make sure that the future FOCs are consistent with the guess. Now notice that the FOCs for $c_2^i$ and for $c_1^i$ are almost identical, with the exception given by the term involving $\lambda_{t-1,3}$: the latter includes $\lambda_{0,3}^i$ while the former does not (only $\lambda_{1,3}^i$ is present). Thus, if we include an extra set of restrictions, $\lambda_{0,3}^i = \lambda_{1,3}^i \forall i$, the two FOCs are identical under the guess. These additional $I$ independent equations complete the full system. Moreover, the FOCs for leisure in the two periods, which also involve $\lambda_{t-1,3}$, are now also identical under the guess. Finally, it is easy to see that all future optimality conditions are met under the guess.

References


