

SROLF\ GHVLJ Q I RU PRGHOV Z LWK UDWLQRQDO H[SHFWDWLRQV DQG OHDUQLQJ

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ABSTRACT. In this paper we present a method for using rational expectations in a linear-quadratic optimization framework with learning. We present a method that allows a policy maker to derive an optimal policy in the presence of rational expectations and the possibility of parameter drift.

41 YA|i Na W| NA

Vwdwqlqj lq wkh odwh 4<: 3*v wkhuh zdv pxfk fulwflvp rq wkh xv h ri frqwuro wkhru | lq hfrqrpplfv lq wklv ylhz/ rqh ri wkh pdmru gudzedfnv zdv wkdw lw frxug qrw ghdo z lwk udwlqrqdo h{shfwdwlrqv +UH,1 Dqrwkhuh gudzedfnv zdv wkh *Lucas critique* lq zklfk d rswlpdo srolf| lv kdpshuhg e| sdudphwhu fkdqj hv gxx wr srolf| dqwlflsdwlrq lq wklv sdshu zh wu| wr plwjdwh erwk gudzedfnv e| lqwurgxflqj d Olqhdutxdgudwlf srolf| iudphzrun wkdw doorzv iru UH dqg ohduqv derxw sdudphwhu guliw gxx wr dqo wflsdwhg srolf| fkdqj hv l

Wkhuh kdv ehq d uhfhqw uhylydo ri lqwhuhvw lq ohduqlqj xqghu wkh wlvoh ri erxqghg udwlqrqdo lw/ Pdufhw dqg Vduj hqw +4<; <, dqg Vduj hqw +4<<6,1 Hduolhu zrurv rq ohduqo lqj lq pdfurhfrqrpplfv lqfoxghv vwxglhv e| Suhvfrww +4<: 5,/ PdfUhd +4<: 5,/ Fkrz +4<: 8, dqg Nhqgulfn +4<; 4,1 lq wklv sdshu zh dssol vlplodu surfhg xuhv xvlqj d Olqhdutxdgudwlf +OT, srolf| iudphzrun lq wkh suhvhqfh ri udwlqrqdo h{shfwdwlrqv lq Z h uvw xv h wkh phwkrq sursrvhg e| Vlpv +4<<9,/ zklfk lv dq h{whqvlrq ri wkh Eodqfkdug dqg Ndkq +4<; 3, phwkrq/ wr vroyh wkh udwlqrqdo h{shfwdwlrqv lq rxu rswl0 pl}dwlrq prghol Diwhu vroylqj wkh wkh udwlqrqdo h{shfwdwlrqv/ lw lv srvleoh wr dssol vwdqgdug frqwuro wr frpsxwh wkh vhw ri rswlpdo lqvwuxphqwv dqg ohduq wkh xqqrzq sdudphwhuv lq wkh prghol

51 i NO,j 6 t|B|j 6j A| BAa tN, | NA

I roorzlqj Nhqgulfn +4<; 4,/ wkh vwdqgdug vlqj oh0dj hqw vwrfdkwlf olqhdutxdgudwlf rswlp}dwlrq sureohp lv zulwwhq dv=

I lqg wkh vhw ri dgplwleoh lqvwuxphqwv $U @ \{u_0, u_1, \dots, u_{T-1}\}$ wkdw plqlpl}hv wkh zhoiduh orw ixqfwlrq

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$$J_T @ E \left\{ \beta^T L_T + x_T, \sum_{t=0}^{T-1} \beta^t L_t + x_t, u_t, \right\} \quad +4,$$

zlwk

$$\begin{aligned} L_T @ & \frac{4}{5} + x_T - x_T, 'W_T + x_T - x_T, \\ L_t @ & \frac{4}{5} + x_t - x_t, 'W_t + x_t - x_t, \\ & \frac{4}{5} + u_t - u_t, 'R_t + u_t - u_t, \cdot + x_t - x_t, 'F_t + u_t - u_t, \end{aligned}$$

vxemhfw wr wkh prgho

$$x_{t+1} @ A + \theta, x_t \cdot B + \theta, u_t \cdot C + \theta, z_t \cdot \epsilon_t \quad +5,$$

Wkh yhfwrw $x_t \in \mathbb{R}^n$ lv wkh wdwth ri wkh hfrqrp| dw wlp t d qg wkh yhfwrw $u_t \in \mathbb{R}^m$ frqwdlqv wkh srolf| lqvwuxphqwv¹ Wkh lqlwdo wdwth ri wkh hfrqrp| $x_0 \in \mathbb{R}^n$ lv nqrzq/ $x_t \in \mathbb{R}^n$ d qg $u_t \in \mathbb{R}^m$ duh wdujhw ydoxhv¹ $W_t \in \mathbb{R}^{(n \times n)}$ / $R_t \in \mathbb{R}^{(m \times m)}$ d qg $F_t \in \mathbb{R}^{(n \times m)}$ duh shqdow| pdwulfhv¹ $\epsilon_t \in \mathbb{R}^n$ lv d zklwh qrlvh yhfwrw zlw k $\epsilon_t \sim N(0, \Sigma)$, $\Sigma > 0$ Zh dwxph wkdw $\epsilon_t \in \mathbb{R}^{(n \times n)}$ lv nqrzq wr wkh srolf| pdnhu¹ Ohduqlaj lv lqwurgxfhg lqwr wkh OT iudphzrun e| wkh xqnrzq sdudphwhu yhfwrw $\theta \in \mathbb{R}^p$ zklfk lv ghwhuplqhg wkurxjk d ohduqlaj vwudwhj |¹

Wkh deryh prgho lv vwudwhj kwiruzdug wr vrojh/ vhh Nhqgulfn +4<; 4,1 Krzhyhu/ d vhlurxv gudzedfn iru hfrqrp|lv lv wkdw htxdwlrq +5, grhv qrw doorz iru UH1 Rqh zd| ri doorzljaj UH wr hqwhu wkh prgho lv wr dxj phqw htxdwlrq +5, lq wkh iroorzljaj idvklrq

$$x_{t+1} @ A + \theta, x_t \cdot B + \theta, u_t \cdot C + \theta, z_t \cdot \sum_{j=1}^k D_j + \theta, E_t x_{t+j} \cdot \epsilon_t \quad +5d,$$

zkhuh wkh pdwul{ $D_j + \theta$, lv d sdudphwhu pdwul{ $E_t x_{t+j}$ lv wkh h{shfwhg wdwth iru wlp t . j dv vhhq iurp wlp t / d qg k lv wkh pd{lpxp ohdg lq wkh h{shfwdwlrq irupdwlrq¹

Lq rughu wr frpsxwh wkh dgplvleoh vhw ri lqvwuxphqwv zh kdyh wr holplqdwh wkh udwlrqdo h{shfwdwlrq iurp wkh prgho¹ Lq dq hduolhu sdshu/ Dppdq d qg Nhqgulfn +4<<: / zh ghvfulehg krz wkh frqwuro prgho zlw k UH fdq eh vrojhg xvlaj Vlpv +4<<9, dssurdfk¹ Lq wkh suvhqw fdvh zh hpsor| Vlpv* phwkrq lq d vlxwdwlrq lq zklfk wkhuh lv ohduqlaj¹

Dwxph wkdw zh kdyh dq lqlwdo hwlpdwh ri wkh sdudphwhu yhfwrw θ / zklfk lv θ_0 d qg dq hwlpdwh ri lww frydulqfh pdwul{ $a_0^{\theta\theta}$ ¹ Wkh wuxh ydoxh ri θ lv xqnrzq wr wkh srolf| pdnhu¹

I roorzljaj Vlpv +4<<9, zh fdq uhzulwh wkh v|vwhp htxdwlrq +5d, lq wkh iroorzljaj dxj phqw irup

¹See also Amman (1996).

$$x_{t+1} = A_0 x_t + B_0 u_t + C_0 z_t + D_0 \epsilon_t \tag{6}$$

zkhuh

$$x_{t+1} = \begin{bmatrix} I - D_1 + \theta_0 & -D_2 + \theta_0 & \dots & -D_{k-1} + \theta_0 & -D_k + \theta_0 \\ I & 3 & \dots & 3 & 3 \\ 3 & I & \dots & 3 & 3 \\ \vdots & & 1_{11} & 3 & 3 \\ 3 & \dots & & I & 3 \end{bmatrix} x_t + \dots \tag{7}$$

$$x_{t+1} = \begin{bmatrix} A + \theta_0 & 3 & 3 & \dots & 3 \\ 3 & I & 3 & \dots & 3 \\ 3 & 3 & I & & 3 \\ \vdots & \vdots & & 1_{11} & \\ 3 & 3 & 3 & & I \end{bmatrix} x_t + \begin{bmatrix} B + \theta_0 \\ 3 \\ \vdots \\ 3 \end{bmatrix} u_t + \begin{bmatrix} C + \theta_0 \\ 3 \\ \vdots \\ 3 \end{bmatrix} z_t + \begin{bmatrix} I \\ 3 \\ \vdots \\ 3 \end{bmatrix} \epsilon_t$$

dqg wkh dxj hphqwng vwdwh yhfwr

$$x_t = \begin{bmatrix} x_t \\ E x_{t+1} \\ E x_{t+2} \\ \vdots \\ E x_{t+k-1} \end{bmatrix} \tag{8}$$

Wdnlqj wkh j hqhudol}hg hj hqydoxhv ri htxdwlrq (6), doorzv xv wr ghfrpsrvh wkh v|vwhp pdwulfhv x_t dqg x_{t+1} lq wkh irrorzljaj pdqghu/ vhh Prohu dqg Vwhzdww +4<: 6, dqg Frhpdq dqg Ydq Ordq +4<; ; /

$$\begin{aligned} & @ Q_0 Z \\ & @ Q_1 Z \end{aligned}$$

zlwk $Z'Z @ I$ dqg $Q'Q @ I$ Wkh pdwulfhv w_t dqg w_{t+1} duh xsshu wuldaj xodu pdwulfhv dqg wkh j hqhudol}hg hj hqydoxhv duh $\forall i \omega_{i,i} / \lambda_{i,i} > 1$ Li zh xv h wkh wudqviruspdwlrq $w_t @ Z'x_t$ zh fdq zulwh htxdwlrq (6), dv

$$w_{t+1} = Q_1 w_t + Q_2 u_t + Q_3 z_t + Q_4 \epsilon_t \tag{9}$$

J lyhq wkh wuldaj xodu vwuxfwxuh ri w_t dqg w_{t+1} zh fdq sduwlrq (7), dv irrorzv

$$\begin{bmatrix} 11 & 12 \\ 3 & 22 \end{bmatrix} \begin{bmatrix} w_{1,t+1} \\ w_{2,t+1} \end{bmatrix} = \begin{bmatrix} 11 & 12 \\ 3 & 22 \end{bmatrix} \begin{bmatrix} w_{1,t} \\ w_{2,t} \end{bmatrix} + \dots$$

zkhuh wkh xqvwdeoh hlj hqydoxhv duh wkh lq orzhu ulj kw fruqhu/ lth lq wkh pdwulfhv
 $\gamma_t @ w_{2,t} @ - \sum_{j=0}^{\infty} M^{j-1} \frac{-1}{22} Q_{2+} u_{t+j} \cdot 3z_t,$ +;

zlwk

$$M @ \frac{-1}{22} 22$$

Uhlqvwulqj htxdwlrq +; , lqwr htxdwlrq +; , j lyhv xv

$$w_{t+1} @ w_t \cdot 2u_t \cdot 3z_t \cdot 4\epsilon_t \cdot \gamma_t \quad +< ,$$

zlwk

$$\begin{aligned} @ \begin{bmatrix} 11 & 12 \\ 3 & I \end{bmatrix} @ \begin{bmatrix} 11 & 12 \\ 3 & 3 \end{bmatrix} 2 @ \begin{bmatrix} Q_1 \\ 3 \end{bmatrix} 2 \\ 3 @ \begin{bmatrix} Q_1 \\ 3 \end{bmatrix} 3 4 @ \begin{bmatrix} Q_1 \\ 3 \end{bmatrix} 4 \gamma_t @ \begin{bmatrix} 3 \\ \gamma_t \end{bmatrix} \end{aligned}$$

Nqrzljaj wkdw $x_t @ Z'w_t$ zh fdq zulwh htxdwlrq +; , dv

$$x_{t+1} @ Ax_t \cdot Bu_t \cdot Cz_t \cdot \epsilon_t \quad +43,$$

zlwk

$$A @ Z^{-1} Z' \quad B @ Z^{-1} 2 \quad C @ [Z^{-1} 3 \quad Z^{-1}] \quad +44,$$

dqg

$$^{-1} @ \begin{bmatrix} -1 & -1 \\ 11 & I \end{bmatrix} z_t @ \begin{bmatrix} z_t \\ \gamma_t \end{bmatrix} \epsilon_t @ Z^{-1} 4\epsilon_t \quad +45,$$

Lq qrq0sdwkrorj lfd0 fdvhw/ Vwhzdww +4<: 6,/ wkh pdwul{ 11 zloo eh qrqvlaj xodu1
 Z lwk htxdwlrq +43, zh kdyh vroyhg wkh UH1 sduw ri wkh frqwuro prgho1

$$61 C j , j Bi A A \} B, \} Ni | 6$$

Dv lq htxdwlrq +5,/ wkh pdwulfhv $A/ B/ C$ ghshqg rq wkh xqqrzq sdudphwhu
 yhfwrw θ dqg zh kdyh lqvwuhg dq lqlwdo hwlpdwh θ_0 ri wklv sdudphwhu yhfwrw lq
 rughu wr eh deoh wr vroyh wkh UH1 Wkh yhfwrw ϵ_t dovr ghshqg rq $\theta/$ vr zh kdyh wr
 dvvxp wkdw $E\epsilon_t @ 31$

Krzhyhu/ wkh hwlpdwh θ_0 zloo fkdqjh ryhu wlpdwh dv qhz lqirupdwlrq ehfrphv
 dydlodeoh1 I xuwkhupruh/ srolf| dqqrxfhphqww pd| lq xhqfh wkh xqghuo| lqj ydoxh
 ri wkh wuxh sdudphwhu $\theta/$ Oxfdv +4<: ,1 Khqfh/ dv vrrq dv zh kdyh lpsohphqwhg wkh

²Also see Amman and Kendrick (1997), Appendix B.

uvw frqwuro u_0 / zh zloo jhw d qhz uhdolvdwlrq ri wkh vwdwh yhfwrw x_1 / zklfk hqdeohv xv wr uhhwlpdwh wkh sdudphwhu yhfwrw rewdlqlqj θ_1

Lq wkh olwhudwxuh d qxppehu ri surfhgxuhv iru vxfk ohduqlqj surfhwhv duh ghvfulehg1 I ru lqvwddqfh/ ruglqdu| ohdvw vtxduhv ohduqlqj/ owhulqj/ ru wvrfkdvwlf dssur{lpdwlrq/ Vduj hqw +4<<6,1 Khuh zh zloo dssol d wvrfkdvwlf dssur{lpdwlrq whfkqltxh1 Lq lww prvw vlpsoh irup wkh sdudphwhu yhfwrw θ kd v wr eh wkh ghulyhg iurp wkh olqhdv prgho

$$y_{t+1} @ H_t \theta . \nu_t \quad +46,$$

zkhuh wkh yhfwrw y_{t+1} frqwdlqv wkh hohphqw ri wkh vwdwh yhfwrw x_{t+1} wkdw duh lqyrohg lq wkh hwwlpdwlrq surfhv1 Wkh pdwul{ H_t lv ghvfulehg lq vlpodu whupv1 H_t frqwdlqv wkh gdw +uhjuhvruv, qfhvvdv| wr hwwlpdwh θ_t dqq wkh huuru whup lv $\nu_t \sim N+3, \nu, 1$ I roorzlqj Omxqj dqq Vrguuvurp +4<6, / dv dssolhg e| Vduj hqw +4<<6, / rxu sdudphwhu yhfwrw fdq eh uhhwlpdwhg e| xvlqj wkh iroorzlqj Ed|hvldq xsgdwlrq vfkph iru wkh sdudphwhu hwwlpdwhv

$$\theta_{t+1} @ \theta_t . \alpha_t + a_t^{\theta\theta},^{-1} H_t y_{t+1} - H_t' \theta_t, \quad +47,$$

$$a_{t+1}^{\theta\theta} @ a_t^{\theta\theta} . \alpha_t + H_t' H_t - a_t^{\theta\theta}, \quad +48,$$

Lq wkh deryh htxdwlrqv $a_t^{\theta\theta}$ lv wkh hwwlpdwh ri wkh frydulqfh pdwul{ ri sdudphwhu yhfwrw θ dqq α_t lv dq dgmxxvphqw sdudphwhu³¹

Vdwulqj zlwkw rxu lqlwdo hwwlpdwhv θ_0 dqq $a_0^{\theta\theta}$ zh fdq xsgdwh wkh sdudphwhu yhfwrw hdfk wlpq qhz lqirupdwlrq rq wkh vwdwh ri wkh hfrqrp| ehfrphv dylodeoh1 Li zh frpelq wkh UH vroxlwlrq phwkrq zlwkw ohduqlqj vwuwhj | zh jhw wkh iroorzlqj doj rulwkp

Vwhs 31 Vhw $t @ 3$ dqq frpsxwh dq hwwlpdwh ri θ_t dqq lww fruuhsrqqqlqj frydul0 dqfh pdwul{ $a_t^{\theta\theta 1}$

Vwhs 41 Vhw wkh lwhudwlrq frxqwhu $\nu @ 31$

Vwhs 51 Vhw wkh lqvwuxphqw $u_i^\nu / i @ \{t, t . 4, \dots, T . s - 4\}$

Vwhs 61 Frpsxwh $\gamma_i^\nu / i @ \{t, t . 4, \dots, T . s - 4\}$ dqq frpsxwh A / B dqq C

Vwhs 71 Dssol vwdqgdug OT rswlp|dwlrq phwkrq wr frpsxwh d qhz vhw ri rswlpdo lqvwuxphqw $u_t^{\nu+1}$ xvlqj wkh htxdwlrq ehorz lq sodfh ri htxdwlrq +5d,

$$x_{t+1}^{\nu+1} @ A + \theta_t, x_t^{\nu+1} . B + \theta_t, u_t^{\nu+1} . C + \theta_t, z + \theta_t, \nu$$

Vwhs 81 Vhw $\nu @ \nu . 4$ dqq jr wr Vwhs 5 xqwo frqyhuj hqfh lv uhdvkhg rq wkh UH sdw

³A formal derivation of the above procedure can be obtained from the authors.

Vwhs 91 Hwwlpdwh θ_{t+1} dgg $a_{t+1}^{\theta\theta}$

Vwhs :1 Vhw $t @ t$. 4 dgg jr wr Vwhs 4 li $t < @ T$

Khqfh/ Vwhs 4 wr 8 rxwolqh wkh phwkrq iru vroylqj wkh OlqhdudTxdgudfwlf iudpho zrun iru wkh UH sduw1 Vwhs 9 frqwdlqv wkh ohduqlqj sduw

71 A j ! B6V, j

Lq wklv vhwlrq zh zloo suhvhqwg dq h{dpsoh ri wkh doj rulwkp ghvfulehg lq wkh suho ylrsv vhwlrq1 Wkh uvw sduw ri wkh h{dpsoh frqfhuqv wkh ohduqlqj ri dq xqnrzq exw frqvwdqw sdudphwhu yhfwrul Wkh vhwlrq sduw dgguhwhv wkh Oxfdv fulwtxh lq wkh vhwlrq wkdw zh ohduq dq xqnrzq sdudphwhu yhfwrul/ zklfk lv lq xhqfhg e| ydoxh ri wkh srolf| lqvwxphqwwl

Frqvlghu d vlpsoh pdfur prgho zlw rxwsxw/ x_t / frqvxpswlrq/ c_t / lqyhwphqww/ i_t / j ryhuqphqww h{shqglwxuhv/ g_t / dgg wd{hv τ_t 1 Wkh sureohp fdq wkhq eh vwdwhg dv=

I lgg iru wkh iru wkh prgho

$$\begin{aligned} x_{t+1} & @ c_{t+1} \cdot i_{t+1} \cdot g_{t+1} & +49, \\ c_{t+1} & @ 3.5x_t - \tau_t \cdot 533 & +4, \\ i_{t+1} & @ 3.5E_t x_{t+2} \cdot 433 \cdot \epsilon_t & +4, \\ g_{t+1} & @ u_t & +4, \\ \tau_{t+1} & @ 3.58x_{t+1} & +53, \end{aligned}$$

zlw $x_0 @ 4833$ / d vhw ri dgpwlweoh frqwuro $U @ \{u_0, u_1, \dots, u_9\}$ wr plqlpl}h wkh zhoiduh orw ixqfwlrq

$$J_T @ \frac{4}{5}x_{12} - 4933,^2 \cdot \frac{4}{5} \sum_{t=0}^{11} \{+x_t - 4933,^2 \cdot g_t^2\} \quad +54,$$

Li zh uhgxfh wkh deryh prgho wr rqh htxdwlrq iru rxwsxw zh j hw

$$x_{t+1} @ 3.9x_t \cdot u_t \cdot 3.5E_t x_{t+2} \cdot 633 \cdot \epsilon_t \quad +55,$$

zklfk ohdgv wr wkh dxj phqwhg v|vwhp

$$\begin{bmatrix} 4 & -3.5 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x_{t+1} \\ E_t x_{t+2} \end{bmatrix} @ \begin{bmatrix} 3.9 & 3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_t \\ x_{t+1} \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 3 \end{bmatrix} u_t \cdot \begin{bmatrix} 633 \\ 3 \end{bmatrix} z_t \cdot \begin{bmatrix} \epsilon_t \\ 3 \end{bmatrix} \quad +56,$$

zkhuh $\forall t z_t @ 4$ dgg zh zloo vhw $\epsilon_t @ 43^{-61}$ Ohwv dvvxph wkdw zh gr qrw nqrz wkh ydoxh ri wkh sdudphwhu ydoxhv 3.9 dgg 3.5/ zklfk fdsxuh wudqvlwlrq h hfw dgg wkh h hfw ri wkh UH lq wkh prgho1 Khqfh/ zh dvvxph wkdw zh kdylh d edg lqilwdo hwwlpdwh

$$\theta_0 @ \begin{bmatrix} 3.8 \\ 3.4 \end{bmatrix} \quad +57,$$

dqg ohwv dvvxph ixuwkhu wkdw lw kdvd yduldqfh hwlwlpdwh

$$a_0^{\theta\theta} @ \begin{bmatrix} 3.6 & 3 \\ 3 & 3.4 \end{bmatrix} \quad +58,$$

zklfk duh erwk dueldulo| fkrvhq1 Dv phqwlrqhg deryh zh zloo dvvxph wkdw wkh wuxh sdudphwhu yhfwrw lv

$$\theta^{old} @ \begin{bmatrix} 3.9 \\ 3.5 \end{bmatrix} \quad +59,$$

D fdvh zklfk lv wkhruhwlfdo lpsruwdqfh lv wkh vlwxdlwlrq lq zkhuh wkh xqnrzq sdudphwhu yhfwrw lv qrw frqwdqg/ exw lq xhqfng e| wkh fkrlfh ri wkh srolf| lq0 wuxphqw1 Oxfdv +4<: 9,/ sdjh 73/ duj xhv wkdw glvfuhwlrqdu| srolf| lv olno| wr eh lqh hfwlyh gxh wr wkh idfw wkdw wkh srolf| lqwuxphqw lq xhqfng wkh wkh sdudphwhu yhfwrw θ / vr lq j qhudo whupv

$$\theta @ G+u_t, u_{t+1}, \dots, u_{T-1}, \quad +5: ,$$

Wkhuhiruh/ lw lv lqwhuhwltaj wr lqyhwljdwh wr zkdw h{whqw rxu frqwuro prgho lv deoh wr ohduq wklv h hfw txfno| hqrjxk wr pdnh glvfuhwlrqdu| srolf| h hfwlyh1 Wr lqyhwljdwh wkh ohduqlaj fdsdelolw| ri wkh doj rulwkp zh zloo dovr dvvxph wkdw wkhuh lv d uhj lph vzlwfk lq shulrg 81 I ru wkh shulrg 8 xs wr 45 zh zloo dvvxph wkdw wkh wuxh sdudphwhu yhfwrw lv

$$\theta^{new} @ \begin{bmatrix} 3.: 3 \\ 3.58 \end{bmatrix} \quad +5: ,$$

E| dssolaj wkh T] idfwrul}dwlrq/ Frohpdq dqg ydq Ordq +4<; ; ,/ zh fdq frpsxwh wkh wkh T] ghfrpsrvlwrq

$$+\theta_0, @ \begin{bmatrix} 4.54; 4 & -3.: 53; \\ 3 & 3.3; 54 \end{bmatrix} \quad +\theta_0, @ \begin{bmatrix} 3.9763 & 3.7; 48 \\ 3 & 3.: : 9 \end{bmatrix} \quad +5< ,$$

$$Z+\theta_0, @ \begin{bmatrix} 3.; ; 77 & -3.799; \\ 3.799; & 3.; ; 77 \end{bmatrix} \quad Q+\theta_0, @ \begin{bmatrix} 3.9; : : & 3.: 593 \\ -3.: 593 & 3.9; : : \end{bmatrix} \quad +63,$$

vr wkh hlj hqydoxhv duh $\{3.9763/4.54; 4, 3.: : 9/3.3; 54\} @ \{3.85: <, <.7: 54\}$ dqg dssduhqwo| wkh rughulaj ri wkh v|vwhp lv vxfk wkdw wkh xqvwdeoh urrw <.7: 54 lv lq wkh orzhu ulj kw fruqhu1 Wkh rwkhu frpsrqhw duh

$$A+\theta_0, @ \begin{bmatrix} 3.57<9 & 3.85: 4 \\ 3.464; & 3.5: ; 5 \end{bmatrix} \quad B+\theta_0, @ \begin{bmatrix} 3.7<<6 \\ 3.5968 \end{bmatrix} \quad +64,$$

$$C+\theta_0, @ \begin{bmatrix} 47<.: : <: & 3.: 593 & 3.3898 \\ : <.3966 & 3.6; 65 & 4.4939 \end{bmatrix} \quad +65,$$

$$W_t @ \begin{bmatrix} 4 & 3 \\ 3 & 4 \end{bmatrix} \quad R_t @ [4] \quad F_t @ \begin{bmatrix} 3 \\ 3 \end{bmatrix} \quad +66,$$

$$x_0 @ \begin{bmatrix} 4833 \\ 4833 \end{bmatrix} \quad x_0 @ \begin{bmatrix} 4933 \\ 3 \end{bmatrix} \quad +67,$$

Qrwh/ wkdw zh vhw $E_0 x_1$ htxdo wr 4833 iru wkh uvw lwhudwlrq1 Lq rughu wr ghdo zlwkwkh erxqghu | frqglwlrqv ri wkh UH sduw ri wkh prgho zh qhhg wkh vwhdg | vwdwh ri wkh v|vwhp1 Xqiruwxdwhe/ wkh srlf | pdnhu fdqqrw frpsxwh wkh vwhdg | vwdwh dv wkh vwhdg | vwdwh ghshqgv rq wkh xqnqrzq sdudphwhu yhfwrw θ

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